THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

A REGULAR meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, October 28, 1899. Thirty persons were in attendance at the two sessions, including the following members of the Society:

Professor Maxime Bôcher, Professor E. W. Brown, Professor F. N. Cole, Professor T. W. Edmondson, Professor A. M. Ely, Professor T. S. Fiske, Mr. A. S. Gale, Dr. G. B. Germann, Miss Carrie Hammerslough, Mr. S. A. Joffe, Mr. C. J. Keyser, Dr. G. H. Ling, Professor F. H. Loud, Dr. Emory McClintock, Dr. James Maclay, Dr. G. A. Miller, Professor E. H. Moore, Professor F. Morley, Professor B. O. Peirce, Professor A. W. Phillips, Professor James Pierpont, Professor M. I. Pupin, Miss Amy Rayson, Professor C. A. Scott, Professor H. D. Thompson, Professor E. B. Van Vleck, Professor A. G. Webster, Miss E. C. Williams, Professor R. S. Woodward.

The President of the Society, Professor R. S. Woodward, occupied the chair. At the opening of the morning session a recess was taken to enable the members to attend the presidential address of Professor H. A. Rowland before the American Physical Society, which was in session in the same building. The simultaneous meeting of the two Societies, which is intended to be a permanent feature, naturally added to the interest and activity on both sides. The two organizations are modelled on a common plan; they have many members in common; and their hearty coöperation, which seems assured, cannot fail to be an important factor in the advancement of science.

The Council announced the election of the following persons to membership in the Society: Professor M. E. Bogarte, Northern Indiana Normal School, Valparaiso, Ind.; Mr. A. S. Gale, Yale University, New Haven, Conn.; Mr. B. F. Groat, University of Minnesota, Minneapolis, Minn.; Dr. Edward Kasner, Columbia University, New York, N. Y.; Professor J. A. Miller, University of Indiana, Bloomington, Ind.; Professor A. M. Sawin, Clark University, Atlanta, Ga.; Professor S. A. Singer, Capital University, Columbus, Ohio; Dr. H. E. Slaught, University of Chicago, Chicago, Ill.; Professor E. P. Thompson, Miami University, Oxford, Ohio. Seven applications for membership were received.

With a view to economy of time, the Council voted that

hereafter the morning session of the meetings of the Society should open at 11 a. m. and the afternoon session at 2 p. m. The Council will meet at 10.15 a. m.

The next meeting of the Society, Thursday, December 28th, will be the annual meeting for the election of officers and three other members of the Council. At this meeting President R. S. Woodward will deliver a presidential address on "The century's progress in applied mathematics."

The following papers were presented at this meeting:

(1) Professor PAUL GORDAN: "Formentheoretische Entwickelung der in Herrn White's Abhandlung über Curven dritter Ordnung enthaltenen Sätze."

(2) Professor E. O. LOVETT: "The transformation of straight lines into spheres."

(3) Dr. G. A. MILLER: "On the simply transitive primitive groups."

(4) Professor C. A. SCOTT: "The conditions imposed on a curve by assigned multiple points."

(5) Professor E. H. MOORE: "On the generational determination of abstract groups" (preliminary communication).

(6) Professor C. A. SCOTT: "The status of imaginaries in pure geometry."

(7) Professor MAXIME BÔCHER: "On Sturm's theorem of comparison" (preliminary communication).

(8) Professor F. MORLEY: "On a fundamental geometric construction."

(9) Mr. E. B. WILSON: "The decomposition of the general collineation of space into three skew reflections."

(10) Dr. G. A. MILLER: On the order of the product of two substitutions."

(11) Mr. J. K. WHITTEMORE: "On a generalization of the fundamental problem of the calculus of variations."

(12) Mr. J. L. COOLIDGE: "A projective representation of the imaginary points of a plane."

Professor Gordan's paper was presented to the Society through Professor H. S. White, and Mr. Coolidge's through Professor Bôcher. Mr. Wilson was introduced by Professor Pierpont. In the absence of the authors the paper of Professor Gordan was read by Professor Moore, that of Professor Lovett was read by title, and those of Mr. Whittemore and Mr. Coolidge were read by Professor Bôcher.

Dr. Miller's first paper appears in the present number of the BULLETIN. Miss Scott's second paper will appear in a

subsequent number. Abstracts of the other papers are given below.

Professor Gordan's paper will appear in the *Transactions*. The following is a summary of its content: The theorems proven by Professor H. S. White in a paper read at the Columbus meeting of this Society in August, 1899, were established by the use of a canonical form of the ternary cubic. These theorems can be obtained quite as readily without using a canonical form. For this purpose the Hessian and Cayleyan are written for the linear combinations $xf + \lambda\Delta$ and $x\pi + \lambda s$, where f denotes a nonsingular ternary cubic, Δ its Hessian, π its conjugate cubic in contragredient variables, and s its Cayleyan, the Hessian of π. The forms so found are cubics in x, λ; hence are determined the parameters of curves having the same Hessian or the same Cayleyan as either f or π. These parameters contain a square root of an invariant of f, but no other irrationality. From explicit formulæ so found it is easy to calculate the simplest invariants of each of the eight forms called A, B, F, G, φ, ψ, α, β in the paper of Professor White, and to show that for certain pairs of them a simultaneous covariant resembling θ_f must be identical with one of the pair multiplied into an invariant factor. These invariant factors are explicitly determined; and the conjugate property of A and α, B and β, F and φ, G and ψ is exhibited. Finally it is shown that a set of three invariantive equations is sufficient to characterize completely such a triple system as f, A, B. The process is in general the converse of that employed in the earlier paper cited above, and confirms its results, with the addition of important details.

Professor Lovett's paper, which will appear in the *American Journal of Mathematics*, is in abstract as follows: The transformations of the point space (x, y, z) into the point space (X, Y, Z) which are determined by the two *æquationes directrices*

$$x\varphi_1 + y\varphi_2 + z\varphi_3 + \varphi_4 = 0, \quad x\varphi_5 + y\varphi_6 + z\varphi_7 + \varphi_8 = 0, \quad (1)$$

where $\varphi_i \equiv a_iX + b_iY + c_iZ + d_i$ (a_i, b_i, c_i, d_i constants),

define a family of ∞^{30} transformations which change the straight line

$$y + kx + m = 0, \quad z + lx + n = 0, \quad (2)$$

into the quadric

$$\begin{vmatrix} k & m \\ l & n \end{vmatrix} \cdot \begin{vmatrix} \varphi_2 \varphi_3 \\ \varphi_6 \varphi_7 \end{vmatrix} - k \begin{vmatrix} \varphi_2 \varphi_4 \\ \varphi_6 \varphi_8 \end{vmatrix} - l \begin{vmatrix} \varphi_3 \varphi_4 \\ \varphi_7 \varphi_8 \end{vmatrix} - m \begin{vmatrix} \varphi_1 \varphi_2 \\ \varphi_5 \varphi_6 \end{vmatrix} - n \begin{vmatrix} \varphi_1 \varphi_3 \\ \varphi_5 \varphi_7 \end{vmatrix} + \begin{vmatrix} \varphi_1 \varphi_4 \\ \varphi_5 \varphi_8 \end{vmatrix} = 0. \tag{3}$$

This quadric reduces to a sphere, without k, l, m, n entering the equations of condition, in the following cases and in no other: 1° when any determinant of the matrix

$$\left\| \begin{matrix} \psi_1 \psi_2 \psi_3 \psi_4 \\ \psi_5 \psi_6 \psi_7 \psi_8 \end{matrix} \right\| \qquad (\psi_i \equiv a_i X + b_i Y + c_i Z \equiv \varphi_i - d_i), \tag{4}$$

reduces to the form Const. $(X^2 + Y^2 + Z^2)$, (5)

and at the same time the functions φ_i corresponding to the ψ_i remaining in the matrix reduce to constants; for example, $\varphi_2, \varphi_3, \varphi_6, \varphi_7$ constants and the determinant $\psi_1\psi_8 - \psi_4\psi_5$ of the form (5); 2° when all six determinants of the matrix (4) are of the form (5).

The first case gives six families each of ∞^{13} line-sphere contact transformations determined by two linear equations of the form

$$\begin{aligned} &(a_1 X + b_1 Y + c_1 Z + d_1)x + d_2 y + d_3 z + a_4 X \\ &\qquad + b_4 Y + c_4 Z + d_4 = 0, \\ &(a_5 X + b_5 Y + c_5 Z + d_5)x + d_6 y + d_7 z + a_8 X \\ &\qquad + b_8 Y + c_8 Z + d_8 = 0, \end{aligned} \tag{6}$$

where the constants are subject to the conditions

$$\begin{gathered} a_1 a_8 - a_4 a_5 = b_1 b_8 - b_4 b_5 = c_1 c_8 - c_4 c_5, \\ a_1 b_8 + b_1 a_8 - a_4 b_5 - b_4 a_5 = 0, \quad b_1 c_8 + c_1 b_8 - b_4 c_5 - c_4 b_5 = 0, \\ c_1 a_8 + a_1 c_8 - c_4 a_5 - a_4 c_5 = 0. \end{gathered} \tag{7}$$

Assigning the following particular values to the above constants

$$\begin{gathered} a_1 = a_8 = b_1 = b_8 = c_4 = c_5 = d_1 = d_2 = d_4 \\ = d_5 = d_7 = d_8 = 0, \\ a_4 = a_5 = c_1 = -c_8 = d_3 = d_6 = 1, \\ b_4 = -b_5 = i = \sqrt{-1}, \end{gathered} \tag{8}$$

we have the celebrated correspondence of Lie determined by the equations

$$Zx + z + X + iY = 0, \quad (X - iY)x + y - Z = 0. \tag{9}$$

The above transformations change right lines to points if

$$d_1 : d_2 : d_3 : d_4 = d_5 : d_6 : d_7 : d_8. \qquad (10)$$

The second class of conditions demands that the functions ψ_i have the forms

$$\begin{aligned} \psi_j &\equiv a_j X + i a_j Y + c_j Z, \\ \psi_{j+4} &\equiv c_j X - i c_j Y - a_j Z \end{aligned} \quad (j = 1,2,3,4); \qquad (11)$$

the corresponding equations (1) define a fifteen-parameter family of line-sphere contact transformations whose transformations may be derived by operating on the transformations of the ordinary projective group with the transformation (9) of Lie.

In conclusion the note points out the corresponding generalizations for spaces of higher dimensions which have been incorporated in another paper; sets up a six-parameter family of line-sphere contact transformations by generalizing the form given by Darboux* to Lie's transformation; and finally calls attention to the fact that infinite families† of line-sphere contact transformations have been constructed.

If the nominal conditions imposed on a curve by points of assigned order are in number N, and the actual conditions are in number D, the m-ic excess e_m is $N-D$. The variation in e_m for different values of m has been studied in detail by Dr. Macaulay (*Proceedings of the London Mathematical Society*) for the case where all given points are simple. The object of Professor Scott's paper was to give independent proofs for the case of multiple points. The theorems proved are: (1) if m be sufficiently great, $e_m = 0$; (2) $e_m - e_{m+1} \leq m$, where the sign of equality can occur only in a particular case; (3) $e_m - e_{m+1} \geq 0$, where the sign of equality occurs only if $e_m = 0$.

On the basis of the current definition of abstract groups (discontinuous or continuous, of order finite or infinite) Professor Moore's paper investigates under what conditions and in what sense a set of generator symbols subject to certain generational relations serves to determine a definite abstract group. This paper will be published in the *Transactions*.

*Darboux, Théorie des surfaces, vol. 1, §157.

†*Comptes rendus*, August 21, 1899.

It was the object of Professor Bôcher's communication to show how some of Sturm's theorems concerning linear differential equations of the second order can be proved rigorously and at the same time simply. The analogy of a part of the method here used with a method suggested by Mr. Porter (Cf. BULLETIN, March, 1897, p. 210) should be noticed. Rigorous but complicated proofs had previously been given by the author in the BULLETIN for April, 1898.

Sturm's first "theorem of comparison"* will evidently be proved if it can be shown that y_2 has at least one root in each of the intervals $a<x<x_1$, $x_1<x<x_2$, ···. By combining equations numbered (1) and (2) we readily get

$$y_2(x_1)y_1'(x_1) = \int_a^{x_1}(\varphi_1 - \varphi_2)y_1y_2dx.$$

If y_2 had no root in the interval $a<x<x_1$, y_1 and y_2 would have the same sign throughout this interval. The right side of the above equation would therefore be greater than zero. The left side is, however, obviously either negative or zero. In either case we have a contradiction, and we thus see that y_2 has at least one root in the interval $a<x<x_1$. Consider now the solution $\bar{y}_2$ of (2) which vanishes at x_1 and whose derivative there has the same value as the derivative of y_1 at that point. By a proof exactly like the one just given we see that $\bar{y}_2$ has at least one root in the interval $x_1<x<x_2$, and, therefore, by a well known and easily proved theorem of Sturm,* the same is true of y_2. In the same way each of the other intervals $x_2<x<x_3$, $x_3<x<x_4$, ··· is treated. The second theorem of comparison (*loc. cit.*, p. 300) admits an analogous treatment; and the same is true of certain generalized forms of these theorems.

Professor Morley proved the following theorem: Given three points and four planes, take the polar of any two planes as to the points, and let this polar point be joined to the intersection of the other two planes. We thus have six planes. These meet in a point; and the third polar of this point as to the four planes is the polar of the original three points as to the original four planes. A precisely similar theorem holds for $n+1$ spaces S_{n-1} and n points in a space S_n of n dimensions. That is, the polar of any $n-1$ spaces

* Loc. cit., p. 298. The notation there explained is used here without change.

* Cf. BULLETIN, March, 1897, p. 209.

S_{n-1} as to the points is joined to the intersection of the other two S_{n-1}'s. We have thus $n(n+1)/2$ spaces S_{n-1} which meet in a point; the n^{th} polar of this point as to all the S_{n-1}'s is the polar of the n given points as to all the S_{n-1}'s. This yields, for points and lines lying in a plane, the proof of some statements made by Caporali (Works, Pellerano, Naples, 1888; pp. 262–269).

Mr. Wilson's paper showed that the collineation in three dimensional space can, in general, be resolved into three skew reflections. The method pursued was to give a geometrical construction for three skew reflections whose product was the given collineation. The paper is offered for publication in the *Transactions*.

Dr. Miller showed that if any three arbitrary numbers larger than unity are given we can always find three substitutions s_1, s_2, s_3 whose orders are these three numbers respectively, in the given order, and which satisfy the equation $s_1 s_2 = s_3$. These substitutions can always be so selected that the number of the different elements involved in all of them does not exceed the order of the largest of the given numbers increased by two. In the proof of this theorem the following lemma was frequently used: If two circular substitutions have an odd number of consecutive elements in common their product is a circular substitution that involves all the elements of both.

Mr. Whittemore's paper is in abstract as follows: The fundamental problem of the calculus of variations may be said to be: To determine y, a function of x, whose values for $x = a$ and $x = b$ are given, such that

$$u = \int_a^b F(x,y,y')dx \qquad (1)$$

shall be a maximum or a minimum. In (1) F is a given function, supposed analytic, of x,y, and $y' = dy/dx$. This problem is identical with the following: To determine y, a function of x, whose values for $x = a$ and $x = b$ are given, such that a solution of the differential equation

$$u' = F(x,y,y') \qquad (2)$$

which vanishes for $x = a$, shall have a maximum or a minimum for $x = b$. The second statement leads naturally to the posing of the same problem for the more general differential equation

$$u' = F(x,u,y,y'). \tag{3}$$

If we denote by η the variation of y and by π the corresponding "first variation" of u, and if we suppose η to be a "regular" variation, that is such a variation that η'^2 may be neglected as well as η^2, it follows from (3) that π is determined by the linear differential equation of the first order

$$\pi' - \frac{\partial F}{\partial u}\pi - \frac{\partial F}{\partial y}\eta - \frac{\partial F}{\partial y'}\eta' = 0, \tag{4}$$

where we have further the initial condition $\pi(a) = 0$. A necessary condition that u shall have for $x = b$ a maximum or a minimum is $\pi(b) = 0$ for all regular variations η where $\eta(a) = \eta(b) = 0$. If now in (4) we regard u, y, and η as known functions of x, we can find π by quadratures, and we may then see by the usual method of the calculus of variations that we must necessarily have

$$\frac{\partial F}{\partial y} + \frac{\partial F}{\partial u} \cdot \frac{\partial F}{\partial y'} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0. \tag{5}$$

This condition corresponds to Lagrange's necessary condition for a maximum or a minimum in the calculus of variations, and reduces to it when $\dfrac{\partial F}{\partial u} = 0$.

If now we consider the differential equation

$$u'' = F(x,u,u',y,y') \tag{6}$$

we have for π, the "first variation" of u, a linear differential equation of the second order

$$\pi'' - \frac{\partial F}{\partial u'}\pi' - \frac{\partial F}{\partial u}\pi - \frac{\partial F}{\partial y}\eta - \frac{\partial F}{\partial y'}\eta' = 0, \tag{7}$$

which cannot be solved by quadratures. A method is therefore necessary which does not depend upon the integration of (7). Such a method is discussed in the present paper, and applied to the equation (3) leads again to (5). Applied to (6) the method leads to the proposition: There is in general no function y, having given values for $x = a$ and $x = b$ such that u, determined by (6) and the two initial conditions $u(a) = u_0$, $u'(a) = u_0'$, shall have a maximum or a minimum for $x = b$.

It seems probable that the same method would lead to a similar statement for all differential equations of an order

greater than one in u. The presence in the differential equations of derivatives of y of order greater than one does not affect the result in the latter case, while in the case of equation (3), as in the calculus of variations, we are led to a resulting differential equation more complicated than (5).

Mr. Coolidge's paper began with a definition of imaginary geometrical elements after von Staudt and August. The usual method of representing imaginary points in a real line by real points in a plane was then mentioned, with an explanation why this proceeding could not be so extended as to include all imaginary points in a plane. It was then shown that every imaginary point in a real plane might be represented by the real base of its projection from a real point on an imaginary plane, and the necessary construction was explained. The assemblages of lines representing points on real and imaginary lines were discussed, as well as the relations of those lines which represented chains of imaginary points. The paper closed with a brief reference to the congruences of lines representing points lying on coinc sections, and a discussion of the advisability of finding a method of representing all the imaginary points in space.

F. N. COLE.

COLUMBIA UNIVERSITY.

NOTE ON THE SIMPLY TRANSITIVE PRIMITIVE GROUPS.

BY DR. G. A. MILLER.

(Read before the American Mathematical Society, October 28, 1899.)

THE simply transitive primitive groups present many difficulties which have not yet been solved. In his "Traité des substitutions" Jordan proved some fundamental theorems in regard to these groups.* Several additional theorems appeared in the "*Proceedings of the London Mathematical Society*," 1897, pp. 534–536. The following theorems and corollaries are closely connected with the results of this article.

Every primitive group G of degree n contains a maximal subgroup G_1 of degree $n-1$. When G is simply transitive

*P. 284.

G_1 is intransitive and the order of each of its transitive constituents is divisible by the same prime numbers.†

THEOREM I. *When G_1 is of order p^m, p being any prime number, it cannot contain as a transitive constituent any regular group whose order is less than the order of G_1.*

The subgroup of G_1 which would correspond to identity in this transitive constituent would be self-conjugate and it would have to occur in each of the conjugates of G_1 which do not involve all the elements of the given transitive constituent. In each of these conjugates it would be transformed into itself by substitutions that are not found in G_1, since each subgroup of any group of order p^m is transformed into itself by substitutions that are not contained in it. These conditions are impossible, since G_1 is a maximal subgroup of G.

COROLLARY I. *When G_1 is of order p^m it cannot contain any self-conjugate subgroup of degree less than $n-1$ and of order greater than p^{m-3}, $m>2$.*

COROLLARY II. *When one of the transitive constituents of G_1 is of order p^2, G_1 must be of this order and G must be of class $n-1$.*

THEOREM II. *If the order of G_1 is p^5 and if G_1 contains a transitive constituent of order p^3, then any two of the p^2 conjugates of G_1 which do not involve all the elements of the given transitive constituent contain a common abelian group of order p^3.*

It follows from theorem I that the given transitive constituent must be of degree p^2. Each of the given p^2 conjugates contains a subgroup of order p whose substitutions are commutative to every substitution of the group and whose degree does not exceed $n-1-p^2$. This subgroup must occur in G_1 since the self-conjugate subgroup S of G_1 which corresponds to identity in the given constituent of order p^3 cannot be transformed into itself by any substitution of G except those that are contained in G_1. It cannot occur in S since each subgroup of S is transformed into itself by at least p^4 substitutions of G_1. The given subgroup of order p and S must therefore generate an abelian group of order p^3 which includes all the substitutions of G_1 that are common to it and the corresponding conjugate to G_1.

CORNELL UNIVERSITY,
July, 1899.

† Jordan, loc. cit.

ON THE COMMUTATORS OF A GIVEN GROUP.

BY DR. G. A. MILLER.

LET $s_1, s_2, s_3, \ldots, s_g$ represent all the operators of a given group G, and let t represent any operator whatsoever. From the identities

$$s_\beta^{-1}s_\alpha^{-1}t^{-1}s_\alpha ts_\beta \equiv (s_\alpha s_\beta)^{-1}t^{-1}s_\alpha s_\beta t \cdot t^{-1}s_\beta^{-1}ts_\beta, \quad (\alpha, \beta < g+1)$$

$$s_\beta^{-1}t^{-1}s_\alpha^{-1}ts_\alpha s\beta \equiv (ts_\beta)^{-1}s_\alpha^{-1}ts_\beta s_\alpha \cdot s_\alpha^{-1}s_\beta^{-1}s_\alpha s_\beta$$

we observe that the transform, with respect to any operator of G, of the commutators formed with t and the operators of G is the product of two such commutators. All of these commutators must, therefore, form a group which is transformed into itself by G. When t transforms G into itself the given commutators generate the smallest self-conjugate subgroup of G which has the property that all of the operators of the corresponding quotient group are commutative to t; *i. e.*, t transforms each of the divisions of G with respect to this self-conjugate subgroup into itself. By letting t represent, in succession, all the operators of G we arrive at the known theorem that the commutator subgroup of a group is the smallest self-conjugate subgroup with respect to which the group is isomorphic to an abelian group.*

From the fact that the commutator subgroup of G is a characteristic subgroup it follows that it is self-conjugate in every group that contains G self-conjugately. In particular, if G is of order p^α and is not the abelian group of the type (1, 1, 1, ..., 1), then every group that contains G self-conjugately must also contain a self-conjugate subgroup of order p^β, $\alpha > \beta > 0$. When G is the abelian group of the type (1, 1, 1, ..., 1), it is clearly always possible to construct a group which contains G self-conjugately and which transforms none of the subgroups of G besides identity into itself.†

If t remains fixed in the commutator $C \equiv s^{-1}t^{-1}st$ while s is multiplied on the left by all the operators of G we observe that C remains unchanged when this multiplier is commutative to t and that it is changed for every other multiplier. Arranging all the operators of G in the following

* *Quar. Jour. of Math.*, vol. 28 (1896), p. 268.

† Moore, BULLETIN, vol. 2 (1896), p. 33.

manner, the first row being composed of the operators that are commutative to t,

$$\begin{array}{ccccc} 1, & s_2, & s_3, & \cdots & s_\gamma, \\ r_2, & s_2r_2, & s_3r_2, & \cdots & s_\gamma r_2, \\ r_3, & s_2r_3, & s_3r_3, & \cdots & s_\gamma r_3, \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_\lambda, & s_2r_\lambda, & s_3r_\lambda, & \cdots & s_\gamma r_\lambda, \end{array}$$

C has the same value when s is multiplied on the left by each one of the γ operators of a row, and if s is multiplied by two operators from different rows the corresponding values of C must evidently be different. Hence C has just $\lambda = g \div \gamma$ distinct values when s is multiplied by all the operators of G and each value of C corresponds to the same number of operators of G. This result is independent of whether s or t belong to G or not. When s belongs to G we may say that C has λ distinct values when s is successively replaced by all the operators of G and that each value of C corresponds to the same number of the operators of G.

While C has the same number of different values as t has conjugates when it is transformed by all the operators of G, yet these different values of C need not include any of these conjugates of t nor is it necessary that these form a complete system of conjugates with respect to the operators of G, *e. g.*, when t is one of the operators of order three in the alternating group of order twelve while s assumes successively all the values of the operators of this group, the four values of C are the operators of order two and identity. If s had assumed all the values of the operators of the symmetric group of order twenty-four, C would have assumed the values of four operators of order three in addition to the given four values.

The following two equations

$$\begin{aligned} a_1a_2 \cdot b_1b_2 \cdot c_1c_2 \cdot \cdots \cdot l_1l_2 \cdot m_1m_2 \times a_2b_1 \cdot b_2c_1 \cdot \cdots \cdot l_2m_1 \\ = a_1b_1c_1 \cdots m_1m_2l_2 \cdots c_2b_2a_2, \\ a_1a_2 \cdot b_1b_2 \cdot c_1c_2 \cdot \cdots \cdot k_1k_2 \cdot l_1l_2 \times a_2b_1 \cdot b_2c_1 \cdot \cdots \cdot k_2l_1 \cdot l_2m \\ = a_1b_1c_1 \cdots k_1l_1ml_2k_2 \cdots c_2b_2a_2 \end{aligned}$$

show that every substitution is the product of two substitutions of order two that do not involve any element except those involved in the given substitution.* The given sub-

* It may be observed that each of these substitutions transforms the given product into its inverse.

stitutions of order two are similar when the given cyclical product involves an odd number of letters; when this product involves an even number of letters, one of the given substitutions of order two contains one more transposition than the other. Hence every positive substitution is the product of two similar substitutions of order two. Since these two similar substitutions can be transformed into each other by some substitution we have the theorem:

Every positive substitution is the commutator of two substitutions involving only elements that are contained in the given positive substitution.

From the following examples we see that any circular substitution of an odd order is the product of two similar circular substitutions which may have either one or three common elements,

$$a_1a_3 \cdot a_1a_2 = a_1a_2a_3$$

$$a_1a_2a_3 \cdot a_1a_2a_3 = a_1a_3a_2$$

$$a_1a_3a_4 \cdot a_1a_2a_5 = a_1a_3a_4a_2a_5$$

$$a_1a_2a_3a_4 \cdot a_1a_2a_3a_5 = a_1a_3a_4a_2a_5$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$a_1a_3a_4 \cdots a_{n+2} \cdot a_1a_2a_{n+3}a_{n+4} \cdots a_{2n+1}$$
$$= a_1a_3a_4 \cdots a_{n+2}a_2a_{n+3}a_{n+4} \cdots a_{2n+1}$$

$$a_1a_2a_3 \cdots a_{n+2} \cdot a_1a_2a_3a_{n+3}a_{n+4} \cdots a_{2n+1}$$
$$= a_1a_3a_4 \cdots a_{n+2}a_2a_{n+3}a_{n+4} \cdots a_{2n+1}$$

Hence we observe that any cycle of an odd order greater than three is the product of two smaller similar cycles each of which is of an odd order. It is evident that the two given similar cycles can always be transformed into the inverse of each other by some even substitution; *i. e., every positive cycle whose degree exceeds three is a commutator of two positive substitutions which do not involve any elements except those which are contained in the given cycle.*

From the following equations, in which $0 < 2\alpha < n + 1$,

$$a_1a_2 \cdots a_{n+1} \cdot a_1 a_{n+2} a_{n+3} \cdots a_{2n} a_{2n+1}$$
$$= a_1a_2 \cdots a_{2n} \cdot a_{2n+1} a_{2n+2} \cdots a_{3n}, \qquad n > 1$$

$$a_1a_2 \cdots a_{n+2} \cdot a_1a_2a_3a_{n+3} a_{n+4} \cdots a_{2n} a_{2n+2}$$
$$= a_1a_3a_4 \cdots a_{2n+1} \cdot a_{2n+2} a_{2n+3} \cdots a_{n+2} a_2 a_{n+3} \cdots a_{3n}, \quad (n > 2),$$

we observe that two negative cycles of degrees 2α and $2(n - \alpha)$ are always the product of two positive cycles in-

volving the same elements and of degree $n+1$ if n is even but of degree $n+2$ if n is odd. Since not all the elements of these positive cycles are common, one of them may always be transformed into the inverse of the other by some positive substitution involving the same elements.* We have now proved that every positive substitution which does not include a cycle of order three is the commutator of two positive substitutions that do not involve any element except those of the given positive substitution. If a positive substitution consists of a cycle of order three it is evidently a commutator of positive substitutions with five but no smaller number of elements. If it contains a cycle of order three and some other elements it can clearly be expressed as the product of two positive substitutions such that the one can be transformed into the inverse of the other by a positive substitution in the same elements. Hence

THEOREM I. *Every substitution of the alternating group of degree n $(n>4)$ is a commutator of two substitutions of the same group.*

The inverse of each of the two similar cycles having one common element whose product may be made any desired odd cycle can evidently be transformed into the other by a substitution of any arbitrary order. Hence any odd cycle is the product of two similar substitutions whose order is entirely arbitrary. Similarly we observe that any two negative cycles are the product of two similar substitutions whose order is arbitrary. In particular, any given operator is the product of two operators of the same arbitrary order. Hence the commutators of two operators may be employed to give a very simple proof of this special case of the general theorem. If l, m, n are any integers greater than unity it is always possible to find three operators L, M, N whose orders are l, m, n and which satisfy the relation $L=MN$.

The holomorph of a cyclical group is evidently isomorphic to an abelian group with respect to this cyclical subgroup.† When the order of the given cyclical group is odd the given holomorph must contain systems of conjugate operators each of which includes as many operators as the order of the given cyclical subgroup. When this order is even the number of conjugates in a system must be half as large. Hence

THEOREM II. *If the order of a cyclical group is odd, it is the commutator subgroup of its holomorph and all its operators are*

* Since $ab\cdot cd$ and $abcd\cdot ef$ are the commutators of abc, $ad\cdot bc$ and $bfdec$, $ac\cdot ef$ respectively, we do not need to consider the cases when $n=2$ or 3. In fact, the general method is not directly applicable to these cases.

† Cf. Burnside, Theory of Groups, 1897, p. 240.

commutators of this holomorph. When this order is even, the commutator subgroup of the holomorph includes half of the operators of this cyclical group and all of these operators are commutators of this holomorph.

Since $s^{-1}t^{-1}$ is similar to ts and this is similar to st, we observe that the commutator of two operators is similar to the commutator formed by means of one of these operators and the inverse of the other. The preceding results are, in part, supplementary to those contained in the paper "On the commutator groups," BULLETIN, Vol. IV., pp. 135–139.

CORNELL UNIVERSITY.

THE CALCULUS OF GENERALIZATION.

Calcul de Généralisation. Par G. OLTRAMARE, Doyen de la Faculté des Sciences de l'Université de Genève. Paris, A. Hermann, 1899. 8vo, viii + 191 pp.

THIS work is the magnum opus of the venerable dean of the faculty of sciences, of Geneva, who is probably the oldest living pupil of Cauchy. The volume recapitulates and completes the works of the author published during the last twenty years.

Oltramare regards every function as developable in a series of exponentials; thus, a designating an independent variable, he puts

$$\varphi(a) = A_\alpha e^{\alpha a} + A_\beta e^{\beta a} + A_\gamma e^{\gamma a} + \cdots,$$

where $\alpha, \beta, \gamma, \cdots$ are any constants real or imaginary, in number finite or infinite. He adopts the shorter notation Ge^{au} for the series $\Sigma A_u e^{au}$, u taking successively the values $\alpha, \beta, \gamma, \cdots$, and the equation

$$\varphi(a) = Ge^{au}$$

then expresses that the function $\varphi(a)$ is generated from e^{au} by generalization.

This is in fact an extension of Liouville's generalized derivatives; the latter defined the derivative of index μ of the function $\varphi(a)$ as given by the equation

$$\frac{d^\mu \varphi}{da^\mu} = A_\alpha e^{\alpha a} a^\mu + A_\beta e^{\alpha \beta} \beta^\mu + A_\gamma e^{\alpha \gamma} \gamma^\mu + \cdots;$$

and Oltramare proposed to construct a more general calculus* by considering expressions of the form

* See Laisant's introduction to Oltramare's lithographed essay on the calculus of generalization published previously.

$$A_\alpha e^{\alpha u}\psi(\alpha) + A_\beta e^{\beta u}\psi(\beta) + A^\gamma e^{\gamma u}\psi(\gamma) + \cdots;$$

the functions ψ not containing a; this expression he represents by the symbol $Ge^{au}\psi(u)$, which is clearly the result of an operation effected on $\varphi(a)$ if Ge^{au} represents $\varphi(a)$. Thus, for example, $Ge^{au}u^\mu$ is the μth differential coefficient of the function $\varphi(a)$. Since $e^{au}\psi(u)$ is any function of a and u, $Ge^{au}\psi(u)$ is equivalent to $Gf(u)$, and this last is the result of a certain operation on $\varphi(a)$; this operation is the object of the calculus of generalization; the inverse operation is called degeneralization.

These notions and their usefulness will perhaps be made clearer by a few examples taken from the treatise.

1° Assuming the identity

$$e^{au} = 1 + \frac{au}{1} + \frac{a^2u^2}{1.2} + \cdots + \frac{a^nu^n}{n!} + \cdots,$$

let us define φ by means of the equation

$$Ge^{au} = \varphi(x + a);$$

from this equation we deduce the following

$$G1 = \varphi(x), \; Gu = \varphi'(x), \; Gu^2 = \varphi''(x), \cdots, Gu^n = \varphi^{(n)}(x),$$

hence we have

$$\varphi(x + a) = \varphi(x) + \varphi'(x)\frac{a}{1} + \varphi''(x)\frac{a^2}{1.2} + \cdots + \varphi^{(n)}(x)\frac{a^n}{n!} + \cdots,$$

Taylor's series.

Conversely, putting $\varphi(x)$ equal to e^{xu} we obtain

$$\varphi(x + a) = e^{(x+a)u}, \quad \varphi(x) = ue^{xu}, \cdots, \varphi^{(n)}(x) = u^n e^{xu},$$

which, put in Taylor's series, give the development of e^{au} in series.

2° Let it be proposed to determine the function $\Psi(x, y)$ which satisfies the equation of finite differences

$$\Psi(x, y) - a\Psi(x - 1, y + 1) = 0.$$

We admit that this relation has been obtained by the generalization (using this word always in the technical sense ascribed to it above) of an identity which it is easy to determine.

As every function of two variables can be expressed by the generalizing function Ge^{xu+yv} let us put

$$\Psi(x, y) = Ge^{xu+yv}.$$

This value substituted in the proposed equation gives

$$Ge^{xu+yv}(1 - ae^{-u+v}) = 0;$$

whence, suppressing the factor e^{xu+yv} and degeneralizing, we have

$$1 - ae^{-u+v} = 0;$$

from which relation we deduce

$$e^u = ae^v;$$

raising to the xth power and multiplying by e^{yv} we have

$$e^{xu+yv} = a^x e^{(x+y)v},$$

which generalized gives

$$\Psi(x,y) = a^x \varphi(x + y),$$

the integral of the proposed equation, φ being an arbitrary function.

3° As another example take the integration of the linear equation

$$\frac{dz}{dx} + a\frac{dz}{dy} = bz.$$

Writing $$z = Ge^{xu+yv},$$

which is permissible since every function of two variables can be put in this form, and substituting this value for z in the given equation, the latter becomes

$$Ge^{xu+yv}(u + av - b) = 0,$$

whence, degeneralizing this expression and suppressing the factor e^{xu+yv}, we have

$$u + av - b = 0;$$

and hence $e^{u+av-b} = 1$, or $e^u = e^{b-av}$;

which raised to xth power and multiplied by e^{yv} gives

$$e^{xu+yv} = e^{bx+(y-ax)v},$$

then the value of z will be expressed by generalizing the two members; we shall have

$$z = Ge^{xu+yv} = e^{bx}Ge^{(y-ax)v} = e^{bx}\Psi(y - ax),$$

where Ψ is an arbitrary function.

4° As a last example consider the generalizing function of the numbers of Bernouilli:

$$\frac{ue^u}{e^u-1} = e^{Bu} = 1 + B_1 u + B_2 \frac{u^2}{1.2} + B_3 \frac{u^4}{4!} + \cdots;$$

by replacing u by hu we deduce

$$hue^{hu} = e^{(h+Bh)u} - e^{Bhu},$$

then putting $Ge^{xu} = F(x)$ as defining equation of the function $F(x)$ we have, by generalizing both members of this identity,

$$F(x+h+Bh) - F(x+Bh) = hF'(x+h),$$

the fundamental formula of the theory of Bernouillian numbers.

By multiplying both numbers of the identity above by $e^{xu} - 1$ we have

$$\frac{ue^u(e^{xu}-1)}{e^u-1} = e^{(u+B)u} - e^{Bu} = u(e^u + e^{2u} + \cdots + e^{xu});$$

generalizing both members and putting $Ge^{xu} = f(x)$, we find the summation formula of Maclaurin

$$f'(1) + f'(2) + \cdots + f'(n) = f(n+B) - f(B).$$

These examples show the part to be played by the calculus of generalization when the operations are linear; on the other hand, when the operations are non-linear the calculus is no longer applicable; it may then be regarded as a theory of linear operations.

Its general problem as regards its own mechanism is to determine the value of the expression $G\Psi(u, v, w, \ldots)$ represented by the development of

$$\Psi(D_x, D_y, D_z, \ldots)\,\varphi(x, y, z, \ldots),$$

the function φ being defined by the relation

$$\varphi(x+a, y+b, z+c, \ldots) = Ge^{au+bv+cw+\cdots}$$

The order followed in the construction of the various forms $G\Psi$ and the applications to be made of these tools appear from the chapters of Oltramare's work which are in order 1° calculus of generalization, 2° generalization (technical use of word always) of functions of a single variable, 3° generalization of functions of several variables, 4° of rational functions, 5° of exponential functions, 6° of logarithmic functions, 7° of circular functions, 8° of various forms of transcendental functions, 9° expression of the integral $\int\varphi(x)dx^n$ as a definite integral, 10° differen-

tiation and integration with fractional indices, 11° transformations of series into definite integrals and reciprocally, 12° expression of the sums of certain general series as definite integrals, 13° integration of equations, 14° determination of a particular integral of every linear differential, difference, or partial differential equation, having constant coefficients and second member, 15° inverse calculus of definite integrals, 16° integration of linear differential or difference equations with constant coefficients, 17° integration of linear partial differential or difference equations with constant coefficients, 18° integration of certain partial differential equations with variable coefficients, 19° integration of equations of mixed differences, 20° integration of simultaneous equations, 21° integration of certain equations which can be transformed into linear equations.

One of the principal advantages which the author claims for the method is the application which can be made of it to the integration of differential equations. Its rôle in higher analysis he likens to that of logarithms in numerical reckoning. It makes the integration of equations, difference and partial, single or simultaneous, depend on the solution of algebraic equations; moreover, it permits of determining in general the maximum to the number of arbitrary functions entering an integral in order that it be complete, and in particular cases the process can determine the exact number of these arbitrary functions.

The general process employed by this calculus for the integration of equations consists in representing the known and unknown functions under the generalized forms, substituting these in the equations, and deducing the integrals by the aid of generalization.

E. O. Lovett.

Princeton University.

SHORTER NOTICES.

Jacob Steiner's Vorlesungen über synthetische Geometrie. Zweiter Theil: Die Theorie der Kegelschnitte gestützt auf projective Eigenschaften. Auf Grund von Universitätsvorträgen und mit Benutzung hinterlassener Manuscripte Jacob Steiner's. Bearbeitet von Heinrich Schröter. Dritte Auflage, durchgesehen von Rudolf Sturm. Teubner, Leipzig, 1898. xvii + 537 pp. Price, 14 marks.

The fact that this new edition of Steiner's lectures, edited and published by Schröter in 1866, has been prepared by

Professor Sturm at the request of the publishers, shows that the demand which called for a second edition in 1876 has not been satisfied by the appearance of more modern books on the subject. It is a curious coincidence that Reye's "Geometrie der Lage" also made its first appearance in 1866 and went into a new edition in 1876.

To those not familiar with the scope of the work a short sketch of the contents may be of interest. It was Steiner's intention to publish his investigations in synthetic geometry in five parts, and the first, the important work: "Systematische Entwickelung der Abhängigkeit geometrischer Gestalten von einander" was published in 1832. The remaining four parts never appeared, but much of the subject matter that was to have formed the fifth part was included in a course of lectures which Steiner delivered several times at the University of Berlin. In 1852 Schröter attended a course of lectures by Steiner "Ueber die neueren Methoden der synthetischen Geometrie" and the notes he took of these lectures supplemented by manuscript notes left by Steiner form the basis of the present volume. In the "Systematische Entwickelung" (1832) Steiner, to avoid being thought too radical, had followed the old method, considering a conic as a section of a right circular cone, although, as he says in 1836, he fully felt that this method was unsuitable for his purpose; in his later unwritten work, part 5, he intended to recapitulate briefly the properties of projective ranges and pencils and from these to deduce his definition of the conic and the theories of involution and reciprocal polars. This plan Schröter has carried out in the first two sections of the work under discussion, following Steiner's arrangement almost exactly. Steiner had come to the conclusion that the use of trigonometry might be avoided in the development of the projective properties of lines and pencils and that this new geometry might, with great advantage, be incorporated with the most elementary geometry. Trigonometrical expressions are used, therefore, only in so far as they are needed for the further development of the subject, but the use of metrical properties is not avoided nor is an endeavor made to build up a logical structure on the ground work of fundamental geometrical axioms. Parallel lines and circles, are introduced without definition, algebra and trigonometry are used freely, the principle of duality is adopted without discussion, and the axes, area, curvature, and other metrical properties of conics are investigated.

The third section, of 178 pages, is devoted to an elemen-

tary treatment of pencils and ranges of conics; the different kinds of conics are discriminated for cases of real and imaginary points and the pencil and range are investigated separately, little use being made of the principle of duality.

In the fourth section, of 112 pages, the polar system of the plane and the net of conics are discussed in some detail.

The book, as a text book, has the defect so often found in books compiled from lecture notes—namely, a lack of continuity in notation. This is especially noticeable in the treatment of fundamental points, which are denoted by four or five different sets of letters in as many articles. Professor Sturm has improved the edition greatly in this respect and has modernized the terminology. There are one or two misprints, for instance on page 132, and there is some slight confusion in the proof given on page 60, but the edition seems on the whole remarkably free from errors.

ISABEL MADDISON.

Compositions d'Analyse, Cinématique, Mécanique et Astronomie, données depuis 1889 à la Sorbonne pour la licence ès sciences mathématiques. Par E. VILLIÉ. Paris, Gauthier-Villars, 1898. 8vo, x + 299 pp.

AN American reader will probably peruse this volume more for the insight it gives as to the standard required from French students just finishing their courses in mathematics, than for the matter it contains. The questions are those set in the final examinations for students who have finished the training which corresponds to a college course here, and who expect to go out immediately and teach. The candidate who can successfully grapple with such problems in a limited time must have had a good preparation and be well equipped, as far as knowledge goes, for the profession of teaching mathematics.

M. Villié has previously published two similar volumes of worked-out problems; the present one contains the solution of questions set since 1889. To American and English students, these volumes will be perhaps superfluous, our text books generally doing what is needed in that direction; in France, so far as may be gathered from their mathematical publications, this part is left to the instructor and the volume before us doubtless serves a very useful purpose. In analysis we have chapters containing solved problems in integration, differential equations, orthogonal

trajectories, radii and lines of curvature, asymptotic and geodesic lines, and imaginary variables. The last will be of assistance to any instructor, consisting as it does, of various well chosen examples in integration round a circuit. In the portions containing kinematics and mechanics we find problems similar to those given by, for example, Routh in his various books. The fourth section contains numerical problems in spherical astronomy.

The admirable clearness of explanation and care of details are to be highly commended.

ERNEST W. BROWN.

Keplers Traum vom Mond. Von LUDWIG GÜNTHER. Leipzig, B. G. Teubner, 1898. xxii + 185 pp.

WE should consider Herr Günther's edition of Kepler's "Dream" rather incommensurate with the importance of his text if it had not evidently been a labor of love on his part. The German translation (the Latin of Kepler is not given) occupies 19 pages, that of Kepler's notes perhaps another 50 pages, the editor being responsible for the rest. The work, or rather scientific romance, is little known and it cannot be said to be of importance except in so far as it shows what stage Kepler had reached in astronomical ideas. Its interest is, in fact, less historical than antiquarian. As a romance it would hardly find many readers at the present day, being chiefly details of the appearance of the sky as seen from the moon. The editor is scarcely fair in comparing it with the well-known tale of "Julius" (!) Verne—the first inspiration of many a schoolboy—which he dismisses with a contemptuous remark. The fullness of the notes on astronomical and other matters will satisfy the most exacting of critics.

ERNEST W. BROWN.

A Short Table of Integrals. By B. O. PEIRCE. Boston, Ginn and Co., 1899. 8vo, 134 pp.

THIS is a revised and much enlarged edition of the author's well-known table of integrals, forming a very useful handbook of formulæ which in many cases are too long and complicated to remember. It constitutes a labor saving volume of considerable value. There are 897 formulæ in all. These include the indefinite integrals of many rational and irrational algebraic and transcendental functions, formulæ of reduction, and the more important definite integrals. There are also numerous auxiliary formulæ, for

example, those arising in trigonometry, the principal relations between the elliptic integrals (Jacobian notation), and series for frequently occurring functions. In the last the author has been careful to state the limits within which the expansions are valid. At the end of the book are several numerical tables which include, beside the usual logarithmic and trigonometric ones, some four place tables of the elliptic integrals and of the gamma functions. In future editions a table of contents might be added.

ERNEST W. BROWN.

NOTES.

ATTENTION is called to the change in the hours of meeting of the AMERICAN MATHEMATICAL SOCIETY. Hereafter the morning session will open at 11 o'clock and the afternoon session at 2 o'clock. The Council will meet at 10:15 a. m.

A NEW list of members of the Society will be issued in January. Forms for furnishing necessary information have been sent to each member, and a prompt response will be of great assistance to the Secretary.

A meeting of the National academy of sciences was held at Columbia University, November 14–16, pure and applied mathematics being represented by Professors Cleveland Abbe, C. S. Hastings, E. S. Holden, A. E. Michelson, Simon Newcomb, Mr. C. S. Peirce, and Professor R. S. Woodward. The following mathematical papers were presented at the meeting: Professor R. S. WOODWARD: "The statical properties of the atmosphere," "A direct proof of the effect on the eulerian cycle of an inequality in the equatorial moments of inertia of the earth;" Mr. C. S. PEIRCE: "The definition of continuity," "Topical geometry in general," "The map-coloring problem."

THE preliminary programme for the November meeting of the London mathematical society announced the following mathematical papers: "Note on Clebsch's second method for the integration of a Pfaffian equation," by MR. J. BRILL; "On the forms of lines of force near a point of equilibrium"; "The reduction of conics and quadrics to their principal axes by the Weierstrassian method of reducing quadratic forms;" and "On the reduction of a linear substitution to a canonical form, with applications to linear differ-

ential equations and quadratic forms, by Mr. T. J. I. BROMWICH; "The abstract group isomorphic to the symmetric group of k letters," by DR. L. E. DICKSON; "On Ampère's equation $Rr + 2Ss + Tt + U(rt - s^2) = V$," by PROFESSOR A. C. DIXON; "Certain correspondences between spaces of n dimensions," by Professor E. O. LOVETT; "The fundamental solutions of the indeterminate relation $\lambda x \geqq \mu y$," by MAJOR P. A. MACMAHON.

At the meeting of the Cambridge philosophical society held October 30th, 1899, a mathematical paper, "On semi-convergent series," was presented by MR. W. MCF. ORR.

BEGINNING with the year 1900, the journal *Bibliotheca Mathematica*, devoted to the history of the mathematical sciences, founded in 1887 by G. ENESTRÖM, of the library of the University of Stockholm, will be published by B. G. Teubner, of Leipzig, under the editorial supervision of M. Enestrom. In the future the form of the journal will be that of the *Mathematische Annalen*, and each volume will contain about five hundred and fifty pages. The editor solicits not only historical articles, but also all items of interest relative to current mathematical activity. Manuscripts may be written in German, French, English or Italian, and acceptable contributions will be accorded honoraria of twenty marks per sixteen printed pages, together with from ten to twenty reprints. The editor's address is Brahegatan 43, Stockholm.

THE publishing house of B. G. Teubner, Leipsic, announces that Vorlesungen über die Geschichte der Trigonometrie, by Dr. A. von BRAUNMÜHL; and Vielecke und Vielflache, Theorie und Geschichte, by Dr. M. BRÜCKNER, are in preparation.

Messrs. Longmans, Green and Company, of New York, are issuing the fourth edition of BURNSIDE and PANTON'S Theory of equations in two volumes; the first volume is ready and contains all that is usually given in elementary works on the theory of equations, together with a short chapter on complex numbers; the second volume is to include those subjects which are more appropriately included under the title of modern higher algebra.

Among the mathematical numbers of Harper's Scientific Memoirs, published by Messrs. Harper and Brothers, of New York, are The second law of thermodynamics, edited by Professor W. F. MAGIE; The law of gravitation, by Pro-

fessor A. S. MACKENZIE; The wave-theory of light, by Professor HENRY CREW. The first of these is already in print.

PARIS FACULTY OF SCIENCES. The following mathematical courses are offered during the first semester of the current academic year, each course occupying two hours per week:—By Professor G. DARBOUX: Fundamental principles of infinitesimal geometry.—By Professor E. GOURSAT: Definite integrals and analytic functions.—By Professor P. APPELL: General laws of equilibrium and of motion.—By Professor H. POINCARÉ: The lunar theory.—By Professor V. J. BOUSSINESQ: Mechanical properties and motions of fluids.—By Professor G. KOENIGS: Kinematics of rigid or deformable bodies, applications to the study of machines.—Unassigned: Elements of analysis and mechanics.—By M. H. ANDOYER: General theory of planetary perturbations. Conferences will be conducted by Messrs. RAFFY, HADAMARD, PUISEUX, ANDOYER, and BLUTEL.

For the second semester the following courses are announced:—By Professor É. PICARD: Partial differential equations.—By Professor E. GOURSAT: Differential equations.—By Professor P. APPELL: General laws of motion of systems; Analytic mechanics; Hydrostatics and hydrodynamics.—By Professor BOUSSINESQ: Theory of waves.—By Professor KOENIGS: Theory of machines.—By M. RAFFY: Differential equations and their applications to mechanics and physics.

CAMBRIDGE UNIVERSITY. Dr. E. J. ROUTH and Mr. A. N. WHITEHEAD are moderators and Messrs. E. G. GALLOP and P. C. GAUL examiners for part I. of the mathematical tripos 1900; the examiners for part II. of the same year are Professor W. BURNSIDE and Messrs. H. F. BAKER, G. T. WALKER and H. M. MACDONALD. At St. John's College, Mr. W. A. HOUSTON, fifth wrangler 1896 and Smith's prizeman, 1898, has been elected to a fellowship in mathematics, and Mr. F. W. B. FRANKLAND, third wrangler in 1897, has received a similar appointment at Clare College.

A LEGACY left by one of its members, S. THOMASO VALLAURI, has enabled the Turin academy of sciences to establish a prize of thirty thousand lire to be conferred on the Italian or foreign scientist who shall have published during the interval between January 1, 1899, and December 31, 1902, the most remarkable and celebrated work on any of the

physical sciences, interpreting the phrase "physical sciences" in its largest sense. The prize will be awarded one year after the expiration of this interval. Neither resident nor non-resident national members of the Turin academy are eligible to the competition. No manuscripts will be received, nor will competing works be returned. The president of the academy is S. GIUSEPPE CARLE and S. A. NACCARI is secretary of the section of the physical sciences, mathematical and natural.

THE subject of Professor L. FUCHS's inaugural address as rector of the University of Berlin for the current year was "The reciprocal relations of pure science and technology."

LORD RAYLEIGH, professor of natural philosophy in the Royal Institution of Great Britain, received the Copley medal at the anniversary meeting of the London Royal Society this year for his investigations in mathematical and physical science.

MR. R. F. MUIRHEAD has been elected president of the Edinburgh mathematical society for the coming year.

Dr. J. B. FAUGHT has been promoted from an instructorship to an assistant professorship at the University of Indiana.

DR. H. LIEBMANN has qualified as privat docent in mathematics at the University of Koenigsberg.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

ANDRÉEV (K. A.). Elementary course in analytic geometry. 2nd edition. (Russian.) Kharkov, 1896. R. 3.20

BIANCHI (L.). Lezioni sulla teoria dei gruppi di sostituzioni e delle equazioni algebriche secondo Galois. *Pisa*, 1899. 8°. 283 pp. Fr. 10.00

——. Vorlesungen über Differentialgeometrie. Autorisierte deutsche Uebersetzung von G. Lukat. 3te (Schluss-) Lieferung. Leipzig, Teubner, 1898. 8vo. Pp. 1–16 and 529–659. M. 4.00

BLUMENTHAL (O.). Ueber die Entwicklung einer willkürlichen Funktion nach den Nennern des Kettenbruches für $\int_{-\infty}^{\infty} \frac{\phi(\xi)d\xi}{z-\xi}$. (Diss.) Göttingen, 1898. 8vo 57 pp.

BOBYNIN (V. V.). The section of mathematics, astronomy and physics at the first nine meetings of the Association of Russian naturalists

and physicians; its objects and activity. Part I: The first, second, third and fourth meetings. (Russian.) Moscow, 1896. 8vo. 169 pp.

BOUSSINESQ (J.). Infinitesimal analysis, translated by A. P. Nenachev. Vol. I: Differential calculus. Part 1: Elementary part. (Russian.) Moscow, 1899. 8vo. 312 pp.

BREMIKER (H.). Sur la transformation des courbes algébriques en général et sur celle des courbes du troisième ordre en particulier (d'après des cours de M. Weierstrass et de M. Bruns). (Progr.) Berlin, 1899. 4to. 19 pp.

CAHEN (A.). Sur la formation explicite des équations différentielles du premier ordre dont l'intégrale générale est une fonction à un nombre fini de branches. (Thèse.) Paris, Gauthier-Villars, 1899. 4to. 99 pp.

CAJORI (F.). Notes on the history of logarithms. (Sonderabdruck aus *Abhandlungen zur Geschichte der Mathematik*, IX, pp. 33–39.) Leipzig, Teubner, 1899. 8vo.

CHELPANOV (G.). The problem of space perception in its relation to the doctrine of à priori and innate ideas. Part I: The idea of space from the point of view of psychology. (Russian.) Kiev, 1896. 8vo. 15 + 387 pp. R. 2.50

DE MORGAN (A.). Elementary illustrations of the differential and integral calculus. New edition. Chicago, The Open Court Publishing Co., 1899. 12mo. 8 + 144 pp. Cloth. $1.00

DOLBNIA (I.). Memoir on the theory of Abelian integrals, presented to the Council of the Mining Institute. (Russian.) St. Petersburg, 1896. 8vo. 155 pp.

ENCYKLOPÄDIE der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Teil 1, Band I, Heft 4: F. W. Meyer, Invariantentheorie (Schluss); C. Runge, Gleichungen, Separation und Approximation der Wurzeln; K. T. Vahlen, Gleichungen, rationale Funktionen der Wurzeln, symmetrische und Affektfunktionen; O. Hölder, Gleichungen, Galois'sche Theorie mit Anwendungen. Leipzig, Teubner, 1899. 8vo. 353–512 pp. M. 4.50

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